TECHNOLOGY, MATHEMATICS, AND PEOPLE: INTERACTIONS IN A COMMUNITY OF PRACTICE

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This paper describes our sociocultural perspective on learning in secondary mathematics classrooms where technology is integrated as a central resource. We propose four roles for technology in relation to student learning: Master, Servant, Partner and Extension of Self. One classroom episode is analysed to reveal the different 'voices' that emerge through the interaction of mathematics, people (students and teacher) and technology. We are using this approach to develop a framework for describing and analysing the characteristics of a classroom community of practice.

The sociocultural orientation that underpins our research emphasises the socially and culturally situated nature of mathematical activity, with learning viewed as a collective process of enculturation into practices of mathematical communities. The classroom as a community of mathematical practice supports a culture of sense making, where students learn by immersion in authentic practices of the discipline. This is distinguished from the constructivist position (Cobb and Bauersfeld, 1995), in which priority is given to the individual construction of understanding, and where social interaction acts principally as a source of cognitive conflict to force the reorganisation of personal mental structures.

Our approach is predicated on three basic assumptions of sociocultural theory: (1) Human action is mediated by cultural tools, and is fundamentally transformed in the process. (2) The tools include technical and physical artefacts, but also concepts, reasoning, symbol systems, modes of argumentation and representation. (3) Learning is achieved by appropriating and using effectively, cultural tools currently recognised and validated by the relevant community of practice. To understand how this appropriation occurs, we make use of Vygotsky's notion of the zone of proximal development in order to examine the social and communicative conditions of learning in classroom settings. In particular, we draw on Bakhtin's theory of voice (Renshaw & Brown, 1998) to investigate interactions between people, technology and mathematics in a senior secondary classroom.

THE CONCEPT OF 'VOICE'

In studying classroom interactions we have found it productive to draw on Mercer's (1995) work on the nature of student talk. *Exploratory talk* is characterised by constructive and critical engagement with ideas presented by 'others' in the group or community. Ideas are challenged and defended by proposing justifications, explanations and alternatives. Claims are publicly accountable, and reasoning, being visible, is therefore open to evaluation and critique. In neo-Vygotskian terms this is associated with the internalisation of social forms and processes. Exploratory talk may be contrasted under the Mercer (1995) construction with *disputational talk* characterised by disagreement, assertion and counter assertion among participants, leading to entrenchment of individual perspectives; and *cumulative talk*, characterised by the compiling and accumulation of ideas leading to uncritical concurrence or consensus. In theorising diversity as it applies within the concept of exploratory talk, it has proved useful to develop Bakhtin's theory of voice (Renshaw and Brown, 1998). This theory emphasises the active, situated and functional nature of speech as employed by various groups. The term *personal voice* is given to words used in conversation that are employed by a wider community of users. So in any individual utterance, a number of voices can be heard—the voice of the speaker, but also voices of other community members who have used similar words or constructions to convey meaning. Then the extent to which a speaker uses such expressions can be used to theorise different levels of performance (e.g. explanation, justification, problem solution) that are accepted as valid by the community. However, not all sources and modes of communication may be equally privileged, and not all 'voices' convey values consistent with community ideals.

We consider a classroom in which the pursuit of mathematical competence and understanding occurs within the framework of our three assumptions underlying the sociocultural perspective. This involves mutual interactions between the teacher and individual students, the total group, and subgroups of students; interaction between individual students and peers involving both cognitive and interpersonal exchanges; individual action and reflection; and interaction between all human participants and artefacts such as text material, and in the context of our special interest, technology as in calculators and computers.

The teacher's 'voice' may be heard in several roles. Firstly, the teacher is a facilitator of mathematical progress in which the 'trace' of her/his membership of a mathematical community will influence strongly the manner in which this role is enacted, through the quality of the mathematics that is demonstrated and facilitated. Secondly, the teacher acts as a custodian of the learning climate in which a productive working atmosphere is maintained in the community. A third avenue for the teacher's 'voice' is through the exercise of pedagogical knowledge that is not specifically content related. This is knowledge applied through such skills as modes of questioning and explanation, achieving student participation, and organising the classroom for learning. This 'voice' will echo the values of professional educators and will interact with those referenced above, for the teacher's 'voice' and utterances are indicative of the extent to which community of practice values are at work, and may of course convey images that are inconsistent with such values. Collective argumentation and dialogue may be curtailed or replaced, for example, by transmissive practices emphasising the unquestioned acceptance of rules and formulae, without opportunity for question or debate. Such 'voices' are heard in many classrooms.

Moving now to peer interaction, we note that interchanges between students provide for the flexible exercise of a variety of mutually reciprocal roles such as tutor, critic, audience, checker and so on. While some research on student-student interaction has examined the effectiveness of assigning such roles to students working in groups, our approach differs in the degree of structure imposed on the peer interactions. Our community of practice view looks at the 'wholeness' of the enterprise. Collaborating mathematicians take different roles at different times with respect to their joint projects, and such decisions are part of the collaborative decision making. Hence students must also be provided with the opportunity to adopt different roles in peer transactions, which will be related to (but distinct from) the content being addressed. Good mathematical practice is modelled by teachers not only in applying mathematical expertise, but also through procedural questioning to make public the choices and actions called for in following a certain solution path. And reciprocally the teacher acts as a coach to highlight strategic and procedural decisions when individual students or groups lead a solution process. In this way, the variety of activities through which clarification is sought are constantly raised and reinforced.

So the 'voices' that can be heard during peer interaction are many. For example, the trace of mathematics as previously learned, understood and passed on, will be audible through the voices of teachers and students in prior learning episodes. Other traces will involve the 'voice' of text material as students articulate its perceived authority. Finally, 'voices' that emerge in relation to technological artefacts are of significant interest for their potential impact on the learning culture.

TECHNOLOGICAL 'VOICES'

We theorise four roles for technological devices such as calculators and computers, in which different 'voices' may be heard.

Technology as Master ("What is it allowing me to do?")

Here the student is subservient to the technology—a relationship that can be induced by either technological or mathematical dependence. If the complexity of usage is formidable, student activity will be confined to those limited operations over which they have technical competence. Alternatively if necessary mathematical understanding is absent, the student is reduced to blind consumption of whatever output is generated, irrespective of its accuracy or worth.

Technology as Servant ("Tell the thing what to do!")

In this role technology is basically used as a reliable timesaving replacement for mental, or pen and paper computations. The tasks of the mathematics classroom remain the same—but now they are facilitated by a fast mechanical aid. Unlike the previous category the user is in control, and 'instructs' the technology as an obedient but 'dumb' assistant. Trust in the reliability of the servant means that the output is regarded as authoritative, although the discerning user will continue to monitor reasonableness of outcome against the possibility of keying errors.

Technology as Partner ("How can we do this together?")

Here a 'rapport' has developed between the user and the technological device—which may be addressed in human terms. A graphics calculator, for example, becomes a friend to go exploring with, rather than merely a producer of results. The user is still in control, but there is appreciation that authority of outcome needs to be judged against criteria other than that a required response has been obtained. Explorations, for example in graphical work, lead to situations where the output needs to be checked against the known mathematical properties of related graphical forms. It is possible for the calculator to be 'wrong', and a feature of use in this mode is the way in which the respective 'authorities' of mathematics and technology are balanced.

Technology as an Extension of Self ("Come fly with me!")

This is the highest level of functioning, and involves users incorporating technological expertise as an integral part of their mathematical repertoire, so that the partnership between student and technology merges to a single identity. Here powerful use of calculators and computers forms an extension of the user's mathematical prowess. Rather than existing as a third party a calculator may be used to share and support mathematical argumentation on behalf of the individual, as when students share and compare computer output as part of their *own* contribution to a solution process. The technology becomes as much a part of the user's catalogue of resources as tabled information and mathematical know-how inside the head.

In a community of practice classroom technology may be involved in any of the latter three roles. At times it will be appropriate to treat technology as a reliable servant for obtaining results necessary to progress a line of development. This may occur frequently during teacher (or student) led group teaching episodes when common output is needed as a focus for discussion. Technology as a partner is important during individual or group exploratory work, and in widening the options in problem solving situations. Technology as an extension of self enters at the higher levels of mathematical activity, where the emphasis is on forms of argumentation, and other characteristics of the discipline of mathematics. Note that there is no necessary connection between these successively higher forms of technological functioning, and the level of mathematics involved, or the seniority of grade level. The 'voices' heard in relation to technology will reflect the mode of use—from despair of a dependent with a flat battery, to instructions given to a servant, through personal conversation with a partner in an investigation, to confident authority in which the contribution of the calculator is merged in the wider argumentation of the user.

RETHINKING THE ZONE OF PROXIMAL DEVELOPMENT IN A COMMUNITY OF PRACTICE SETTING

The different voices that emerge as students appropriate more formal and powerful modes of thinking can be analysed in terms of the Vygotskian notion of the zone of proximal development (ZPD). The most widely known definition of the ZPD is that associated with scaffolding the learning of an individual with help from the teacher or more capable other. However several other contexts have been postulated for the ZPD (Goos, Galbraith, & Renshaw, in press), and in this work we aim to explore the potential of one that addresses learning in the social contexts of classrooms.

The ZPD is normally applied to individuals, but recently it has been applied to whole groups in a way that seems to be both consistent with sociocultural theory, and of practical significance in removing the implication that effective teaching in the ZPD requires sensitive diagnosis of the diverse levels of development of students, followed by one to one instruction (Mercer & Fisher, 1992). Students as participants in a learning community are viewed as having partially overlapping ZPDs that provide a changing mix of levels of expertise that enables many different productive partnerships and activities to be orchestrated (e.g. Brown & Campione, 1995). Through the establishment of a small number of repeated participation frameworks such as teacher-led lessons, peer tutoring, and individual and collaborative problem solving, students become enculturated into taken-for-granted aspects of classroom life that promote a shared knowledge base, a shared system of beliefs, and accepted conventions for communicating and verifying knowledge claims. The lived culture of the classroom becomes in itself a challenge for students to move beyond their established competencies, and adopt the language patterns, modes of inquiry, and values of the discipline.

THE STUDY

We seek to understand and map the changing relationships that characterise classrooms designed on community of practice lines. Complex interactions between students, teacher, the discipline of mathematics, and technology form the context for our study. We aim to identify characteristic behaviours, modes of thinking, and action through the interpretation of classroom events. Included in our range of indicators are the 'voices' identified in the respective dialogues that are a central component of the mathematical exchanges, and the ways that these are elaborated, harnessed, or modified to enhance the quality of learning.

The first year of the study has focussed on a single Year 11 Mathematics C classroom in a co-educational independent school. (Mathematics C is an advanced subject taken by students intending to pursue further study of mathematics at tertiary level.) A series of lessons on matrices was observed and videotaped in order to analyse students' use of technology in learning this new subject matter. Students were also interviewed to seek their interpretations of particular lesson events. Three features of the teaching approach warrant special mention. First, matrix algebra was not taught as a series of algorithms, but instead was developed by presenting students with life related problems from which matrix representation and manipulation arose naturally. Second, students had constant access to graphing calculators to assist them in performing complex matrix calculations. Finally, the teacher regularly asked students to present their solutions to matrix problems by plugging their calculators into an overhead projector panel, so that the whole class could follow the working as it was reconstructed and displayed on the calculator screen.

The following excerpts from a classroom segment (together with commentary) illustrate preliminary attempts to develop a framework consistent with a sociocultural approach to engaging the quality of classroom learning.

Lesson Segment

In this excerpt (in which selected dialogue is labelled in sequence), a student (Paul) leads the development, with other students and the teacher entering the discussion. The problem required students to use matrix multiplication to find the manufacturing costs and wholesale and retail prices of biscuits packaged in different combinations of flavours (see Appendix). The solution being explained has been previously generated on a graphics calculator, and is reconstructed by Paul for display via the overhead projector. The segment begins with another student interposing in a teacher's 'voice', "*Start when you're ready*".

12. P: To start the problem (reads out question (1)) Show the contents of the three proposed packages in a matrix with 3 rows and 4 columns. So I did that with packet name down the rows and the type of biscuit along the columns (Plays with the calculator to get up the screen he wants to show) and there it is, right there!
13. T: Sorry to interrupt! Can I show you another way of doing that? Just type SS1 on the screen, and go to home. Okay, then just clear, so you've got that there. SS1 and then 'enter'. (Students ask questions.)
14. T: You've got to make sure that the calculator knows what you're looking for. So, tell us about what you put in that row. Why is this so?

In this interchange Paul adopts a teacher 'voice' from time to time in displaying his work. "*There it is, right there.*" The teacher intervenes to demonstrate calculator operations, and portrays the calculator in a 'dumb' servant role. "*Make sure that the calculator knows what you're looking for.*" This provides a setting for interpreting the contribution of the calculator.

21. T: Where did SS3 come from?
22. P: Oh, that's just what I was about to say. This! Up here is the number of packets intended to be produced. Down here, will be the type of packets in the same order (inaudible) Yeah! Yeah, I wrote that out later (Teacher and students laugh). I just translated that. Used that T.
23. T: Transposed, Paul! I know what you mean Paul, but it's called Transpose. So, you used 'SS3 transpose'

Here the teacher elaborates a description from Paul who simply "*used that T*". This is not satisfactory from the teacher's viewpoint as a mathematical point remains undisclosed—compounded by the interpretation of 'T' as translate. "*I just translated that.*" The teacher introduces the correct meaning of 'T' as "*transpose*" where its meaning of interchanged rows and columns is evident from the calculator display. This 'voice' of precise mathematics is combined with a 'pedagogical voice' aware of the importance of emphasis: the term '*transpose*' is used three times in a single short sentence. We observe that the use of the term 'translated' by Paul, to describe the change between row and column display, may represent a 'voice trace' from previous mathematics—where 'translate' is associated with movement and changed appearance, as in graphical work.

48. P: So then, in this one, you have the frequency here, of packets produced. You know, the different types of packets. Okay, going back to what I was doing before. Question 4, I did matrices (1) times matrices (2) equal . . .
49. T: Do it, then! (Paul works it out on his calculator)
50. P: Equals that, which costs...which,...These are different types of packs here, and up here, is cost in cents. And I just converted that manually and made the matrices in dollars. Is there any way of

doing that? - tricky, you know! Anyway, I just did that by hand and then created a new one called SS5.

51. T: Okay, well! This is what you could have done, Paul! You could have gone um.... This is something you could show...Just push 'up' a couple of times, again, then 'enter'. So, now, enter that directly into the thing, so that saves you retyping a lot and divide by a hundred Okay, push 'diamond' across to equals. (Students gasp in awe, and Paul nods.) So, what we've done is actually introduce a new type of operation when we're dividing by a number. Can you tell me how that actually works? When you divide it by a number? What does it do? (Students mutter something) Divides what?

52. S's:Divides every number in the result?

53. T: Divides every number in the matrix by that number! And as we've discussed before (inaudible), just multiplying by 100, doing the inverse and that's called scalar multiplication. So, that's two types of multiplication you've seen now. Matrix multiplication which is what you've been doing with these products and scalar multiplication as well. And that's when you multiply and divide through by a constant and in this case, it would be useful to turn cents into dollars.

54. P: I didn't actually divide just the result in dollars. I just did the matrix. If I divided both, I'd get the wrong answer.

Paul rehearses matrix multiplication using the calculator in 'servant' mode and then combines 'hand' arithmetic with 'automatic' calculator processing to create a new matrix, noting an unresolved challenge with respect to the calculator. *"Is there any way of doing that? - tricky, you know!"* The inference is that the calculator *ought* to be able to do it. The teacher now seizes an opportunity to scaffold group learning of the new concept of 'scalar multiplication' in a ZPD enactment. He first achieves a dramatic impact by crashing through calculator operations to produce a graphic display—a significant move that achieves motivation by creating a 'learning gap'. In doing so the calculator is again assumed to be a reliable servant whose output can be accepted. In another sense the calculator is also accepted as an extension of the teacher's trusted expertise—there is no challenge to his button sequence that is incidental to the purpose. The mathematical concept to be learned, 'scalar multiplication', is now explained retrospectively with the benefit of the visible and dramatic end result—an approach not possible if each step must be generated successively by hand. *"So, that's two types of multiplication you've seen now. Matrix multiplication which is what you've been doing with these products and scalar multiplication as well."* Paul rounds off the exchange by relating his mixed mode calculation to the new procedure.

88. T: Why am I hearing so many discussions when Paul is waiting to go on. You should be up there!

89. P: Okay, This SS4 which is each packet named in order and the costs. So, I did SS6 [label for transpose of SS3] times SS4 to work out the amount of money that shops will pay, which is similar to what I got for question 6. Everyone happy with that? After doing, question 7 was pretty similar except I multiplied that by SS7 - this is recommended retail price. SS6 x SS7 gives you the calculation of the total amount that customers will pay for the biscuits. Did everyone have that? And that concludes my demonstration!

Here we hear the teacher's voice adopted also by Paul as he rounds off his demonstration. *"Everyone happy with that?"* ... *"Did everyone have that?"* These queries, while teacher like in tone, do not present the solution as something predetermined and beyond question, but rather represent an invitation to students to submit further challenges so that consensus can be reached. Knowledge is not taken for granted in this classroom: students are expected to offer their work for public critique by peers in order to achieve a communally validated solution.

Student Perspectives

Paul was interviewed soon after the lesson to seek his views on the practice of students presenting their work via the overhead projector. He commented that *"those projector things are good because everyone can see what you're doing"*. Paul also rejected the suggestion that students may be placed under unfair pressure when asked to make their work public in this way, replying *"I think it's good that we should have to get up in front of the class sometimes because, or else you might just never do anything"*. His comments were echoed by other student presenters interviewed after subsequent lessons. For example, students recognised that there is a high degree of accountability associated with public sharing of knowledge. Justification also becomes more authentic when students have to convince their peers that their mathematical arguments are correct. Students agreed that watching a peer explain their work is helpful because it gives them alternative perspectives, the explanation is in language they understand, and they feel reassured when they see someone making the same mistakes they did.

CONCLUSION

While mapping the characteristics of a community of practice culture requires longitudinal study, snapshots reveal attributes in various stages of growth. In global terms we look for this growth developing through processes associated with the classroom culture application of the ZPD as described earlier. These are displayed in the quality and style of student and teacher dialogue *vis a vis* the inclusion of authentic mathematical argumentation such as conjecture, clarification, justification, and interrogation of anomalies. Evidence of such characteristics appear in various forms including 'voice traces' that portray developing expertise appropriated from a variety of sources from which learners have drawn in building their cumulative mathematical know-how. A contemporary interest is the way in which technological artefacts are utilised as resources for learning, and the 'voices' that display the role and status with which they are deployed by teachers and students.

In the lesson discussed we have pointed to several indicators of community of practice attributes. Student talk has reflected the 'teacher's voice' both in giving explanations, and in creating an atmosphere for serious discussion—indicative of a 'partnership' approach in which teacher and learners alternate roles for the purpose of enhancing individual and group learning. Dialogue was driven by the desire to solve but also to understand. This was indicated for example by Paul's asides, in which he nominated puzzles that he felt should be accounted for, and in other excerpts (not included) when other students challenged an interim result. With the teacher's encouragement solution steps were reversed (assisted by the calculator display) until the mistake, a simple transcription error, was located and rectified. Mathematics then, is not seen as a mystery subject, but one in which anomalies are to be resolved, and reasoning is logical, transparent and defensible. Finally some of the 'voices' gave insight into the role that technology was playing in the learning episode. In this segment the graphics calculator was assigned a 'servant' role and used both to obtain results quickly by the students, and for creating a motivational 'learning gap' by the teacher. Through transactions such as the above a classroom culture of inquiry has been developed to act as a ZPD for the cultivation and nurturing of mathematical maturity and expertise. Our interest is in following the potential of such approaches to learning.

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APPENDIX

In planning for Christmas the biscuit factory decided to release three different packets.
Holly Dream (HD): contents 8 chocolate, 12 thin sweet, 6 shortbread, and 4 rich cream.
Sleigh Ringers (SR): contents 12 chocolate, 15 shortbread, and 5 rich cream.
Reindeer Extras (RE): contents 14 chocolate, 16 thin sweet, 12 shortbread, and 8 rich cream.
Manufacturing costs (cents): chocolate 8, thin sweet 4, shortbread 7, rich cream 10.
Packets will be sold to shops for $2.10, $2.95, and $3.70 respectively.
Recommended retail prices will be $2.49, $3.49, and $4.49 respectively.
Numbers of packets to be manufactured (millions) will be: HD (2), SR (1.5), RE (1).

In your workbook answer these questions:
(1) Show the contents of the three proposed packets in a matrix with 3 rows and 4 columns.
(2) Show the manufacturing costs per biscuit in a column matrix.
(3) Show the numbers of each packet to be manufactured in a row matrix.
(4) Use the matrices in (1), (2), and (3) to find the cost to the manufacturer.
(5) Show the prices at which each packet will be sold to shops in a column matrix.
(6) Use your matrices in (3) and (5) to find the total amount the factory charges the shops.
(7) Calculate the total amount which consumers will pay for the biscuits.

Graphics calculator labelling of matrices used by student in dialogue.

$$SS1 = \begin{bmatrix} 8 & 12 & 6 & 4 \\ 12 & 0 & 15 & 5 \\ 14 & 16 & 12 & 8 \end{bmatrix},\ SS2 = \begin{bmatrix} 8 \\ 4 \\ 7 \\ 10 \end{bmatrix},\ SS3 = \begin{bmatrix} 2 \\ 1.5 \\ 1 \end{bmatrix},\ SS4 = \begin{bmatrix} 2.10 \\ 2.95 \\ 3.70 \end{bmatrix}$$